Developing Competences in Future Primary School Teachers under the Conditions of Teacher Education Standardization: A Theoretical Review

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Abstract: Mathematical competence plays an essential role among subject-specific competences of primary school teachers. One should be able to think mathematically, understand mathematical proofs, communicate in “the language of mathematics” and use appropriate codification resources, which makes the concept of mathematical competence rather vital. The concept of primary school teachers’ mathematical competence can be specified through its structure. In this regard, this article analyzes scientific approaches to determining the components of this phenomenon. The structure of future primary school teachers’ mathematical competence is viewed as a system of the following components: motivational-axiological, cognitive, activity-related, communicative, reflexive-creative. It must be noted that the process of developing the structure of this competence and its components, especially content- and activity-related ones, required the authors of the article to analyze regulatory documents and syllabi of the mathematics course of six higher education institutions (HEIs) in Ukraine. All the analyzed syllabi indicate which knowledge, skills and abilities future graduates should possess. The chosen HEIs distribute academic workload differently; at the same time, the content of educational material to be studied is almost identical. The elements of future primary school teachers’ mathematical competence are as follows: arithmetic, algebraic, geometric, that of identical transformations of mathematical expressions, logical.

Keywords: Component structure, content- and activity-related components, academic workload, regulatory documents and syllabi, the mathematics course, components of mathematical competence.

Introduction

Mathematical competence plays an essential role among subject-specific competences of primary school teachers. According to the Recommendation of the European Parliament and of the Council of 18 December 2006 on key competences for lifelong learning, mathematical competence is defined as a key competence that young citizens need for self-fulfilment and life-sustaining activity (Council of the European Union, & European Parliament, 2006). The reference framework of the document sets out eight key competences, including mathematical competence and basic competences in science and technology. In turn, “necessary knowledge in mathematics includes a sound knowledge of numbers, measures and structures, basic operations and basic mathematical presentations, an understanding of mathematical terms and concepts, and an awareness of the questions to which mathematics can offer answers” (Council of the European Union, & European Parliament, 2006, p. 6). Besides, one should be able “to apply basic mathematical principles and processes in everyday contexts at home and work, and to follow and assess chains of arguments; to reason mathematically, understand mathematical proof and communicate in mathematical language, and to use appropriate aids” (Council of the European Union, & European Parliament, 2006, p. 8). Therefore, it is vital to consider the concept of mathematical competence as a key competence.

Interpretation of personal mathematical competence as a component of functional competences in the works of scientists

Some researchers (Gubanova, & Lebedeva, 2009; Ovcharuk, 2007; Melnyk, 2021; Komogorova, 2021; Melnyk, 2019; Maksymchuk, Gurevych, 2020; Maksymchuk, Matviichuk, 2020) understand mathematical competence as an element of functional competences. They claim that the components of these competences are as follows: intellectual development; the ability to apply logic; mathematical knowledge and skills; systemic thinking; the ability to solve complex logical and mathematical problems; spatial thinking; modelling skills. Given such interpretation of mathematical competence, one can define it as a key one since functionality requires one to be willing to apply the obtained knowledge, abilities, and skills to solve professional and life problems.

The emphasis is on the definition of mathematical competence in terms of the subject’s characteristics. As noted by Strilets (2013), this implies a complex combination of one’s mathematical literacy and experience of
mathematical activity, the ability to apply the acquired knowledge in new situations, as well as the striving for self-development. Zinenko (2009) considers mathematical competence as a quality that combines the first two qualities mentioned above.

Mathematical competence directly depends on the level of one’s mathematical activities. The latter is characterized by cognitive activity and achievement motivation; the ability to formulate some real problems in the form of mathematical problems; the ability to find scientifically sound, logical, and rational solutions to mathematical problems; the capacity for self-control and self-analysis of mathematical activity; healthy self-esteem (Strilets, 2013).

Mathematical competence as a key one serves as the basis for professional mathematical competence of teachers, especially primary school teachers. Stelmakh (2011) claims that professional mathematical competence is one’s readiness to apply mathematical tools independently and responsibly to solve professional tasks, as well as system-forming components whose indicators, in the form of mathematical competences, prove theoretical and practical readiness of graduates for professional activities (pp. 11–14).

Rakov (2005) defines mathematical competence of mathematics teachers as “the ability to see and apply mathematics in real life, understand methods of mathematical modelling, build a mathematical, in particular, computer model to study it using mathematics methods reinforced with ICT, interpret the obtained results, estimate calculation errors” (p. 15).

As one can see, mathematical competence of future primary school teachers significantly differs from that of mathematics teachers. Given the research problem of this article, one should pay particular attention to scholars’ views on the concept of mathematical competence of future primary school teachers.

In the context of future primary school teachers’ mathematical culture, Lodatko (2012 p. 18) regards mathematical competence as an element of mathematical culture, especially the ability to apply mathematical knowledge to solve professional problems. The scholar indicates the decreasing quality of future teachers’ knowledge and, therefore, highlights the need to transfer the main mathematical content without simplification.

Gluzman (2010, p. 238) traces the integration of such courses as mathematics (set theory; mathematical statements and their structure; non-negative integers; geometric figures; values), teaching methods of mathematics (teaching methods of arithmetic; teaching methods of geometry; teaching
methods of algebra elements; teaching methods of the problem-solving situation) and teaching technologies of mathematics via a competence-based approach (theoretical and methodical principles of mathematical education of primary school pupils; technologies of teaching mathematics to primary school pupils; a competence-based approach to teaching mathematics).

Given the above, the starting point of Gluzman’s study (2010, p.244) is the concept of methodical-mathematical competence as “one’s systemic characteristic that reflects the integration of theoretical, practice-oriented, research knowledge and skills in mathematics and its teaching methods in primary school, as well as a values-based attitude towards methodical development via self-study, self-fulfilment, socialization, and personal development. Gluzman (2010, p. 152) considers mathematical competence of future primary school teachers as one of the elements of methodical-mathematical competence. In turn, the scholar views mathematical competence of primary school teachers as the ability to use mathematical competencies in the field of theoretical principles of mathematics in primary school at a high professional level, as well as in practice.

The term “methodical-mathematical competence of primary school teachers” is also mentioned in the study by Shmyreva (2013, p. 40). The researcher argues that this competence should manifest itself in a) the knowledge of basic mathematical concepts, laws, and properties offered to primary school children, 2)) the ability to maintain the relationship between concepts and methods of action under study, and c) the ability to formulate mathematical problems following the programme. It is also essential that future teachers should be able to use the obtained knowledge professionally, adapting it to specific learning needs.

However, it was Borzenkova (2007, p.12-13) who first combined mathematical and methodical competences into one (methodical-mathematical competence). According to her interpretation, methodical-mathematical competence is one’s integral characteristic that contains a set of mathematical, methodical, psycho-pedagogical, research competencies, and unity of theoretical, practical, research, and psychological readiness to apply the acquired integrated knowledge and skills when teaching mathematics to primary school pupils. Thus, methodical-mathematical competence acts as an indicator of a teacher’s professional readiness and is realized with the help of the links between its main structural components (procedural, reflexive, personal).
Interpreting the concept of methodological competence, O. Borzenkova (2007) identifies its main characteristics, in particular:

1) knowledge of integrated teaching methods that are based on psychology, pedagogy, mathematics, and its teaching methods;
2) an understanding of pupils’ cognitive activity to organize the educational process effectively;
3) the ability to create optimal learning conditions;
4) the ability to formulate educational tasks and develop fragments of lessons under the basic principles of knowledge acquisition.

The researcher notes the importance of integration of methodological and mathematical disciplines in the training of future teachers of elementary school in high school, believing that only such training can provide effective formation of professional competence of the teacher.

In contrast to Gluzman (2010), Borzenkova (2007) focuses more on methodical competence, almost ignoring its mathematical component. In other words, she considers methodical-mathematical competence of primary school teachers as methodical competence in teaching mathematics. Thus, there is an obvious substitution of the methodical-mathematical competence concept with that of methodical competence.

Skvortsova (2013, pp. 27–36) conducts a detailed analysis of methodical-mathematical, methodical, didactic-methodical competences of future primary school teachers. Methodical competence is understood as “one’s systemic characteristic which manifests itself in the ability to implement and organize mathematics lessons under today’s requirements, as well as successfully solve methodical problems due to theoretical and practical readiness” (Skvortsova, 2013, pp. 35–36).

Similar to Borzenkova (2007), Marko (2014) and Miskova (2013) also identify methodical-mathematical competence of primary school teachers with the teaching of mathematics.

At the same time, Marko (2014) focuses on “an integral characteristic which reflects one’s readiness to carry out professional-pedagogical activities in the field of mathematics and its teaching methods in primary school, as well as to ensure their effectiveness” (p. 113). Besides, the researcher believes it is necessary to know not only concepts and phenomena related to primary education but also to know how to use them to meet primary school pupils’ needs.
Miskova (2013) also wrongly describes mathematical competence of future primary school teachers in the context of methodical-mathematical competence. The researcher considers mathematical competence as a personal quality that lies in the ability to apply the experience of mathematical activity when solving instructional-cognitive, and practice-oriented professional tasks, Miskova (2013, pp. 107–115). Furthermore, Miskova (2013) views such experience as a set of well-developed skills for solving arithmetic, algebraic and geometric problems. Mathematical competence is formed during the study of the mathematics course, as well as related courses.

Shustova (2017, pp. 55-63) analyzes mathematical competence in the context of only one of its aspects, i.e., teacher’s theoretical and practical readiness to carry out effective mathematical activities within the educational process. In turn, such readiness is interpreted as a set of one’s professional and personal qualities to solve problematic situations that may arise during professional activities. Thus, supports the views of Miskova (2013) and considers mathematical competence in the combination of subject-specific mathematical competence and a methodical component of mathematical competence.

According to Baric (2015, p. 95), mathematical competence of future primary school teachers is a component of their professional competence. The researcher emphasizes the teacher’s ability to apply the mathematical apparatus theoretically and practically to perform professional-pedagogical tasks, as well as the integration of subject-specific mathematical and methodical-mathematical competences.

Kazachek (2010, p. 106) defines mathematical competence as one’s integral characteristic that involves profound knowledge of mathematics, abilities to apply the acquired knowledge in new situations and achieve significant professional success. Thus, mathematical competence is a complex combination of essential knowledge and the ability to apply it in practice.

Thus, Baric (2015) and Kazachek (2010), emphasizing mathematical knowledge and the ability to apply it in practice, consider the integration of future primary school teachers’ mathematical competence with methodical one as the basis for teaching mathematics.

Kudryavtsev (2008) also views mathematical competence as one’s integral characteristic based on profound mathematical knowledge and the ability to apply it in practice, only outside methodical competence.
Yevtykhova (2015, pp. 81–85) considers mathematical competence as an element of mathematical literacy and defines it as subject-specific mathematical competence that involves working with numbers and numerical information.

At the same time, V. Panchenko (2013, p. 232) notes that mathematical competence is "an integrative quality that includes a system of motives, psychological qualities, attitudes, professional knowledge, abilities and skills, personal experience that allows the successful implementation of professional activities in the aspect of competence. approach".

Thus, a detailed analysis of future primary school teachers’ mathematical competence shows that some researchers define this competence as actually mathematical (Yevtykhova, 2015; Kudryavtsev, 2008; Honcharuk, 2021; Povidaichyk, 2021; Zhurat, 2020) that implies working with numbers, numerical information and applying them in practice. At the same time, some researchers (Vagis, 2015; Miskova, 2013; Shustova, 2017; Gurevych, 2020; Shahina, 2017; Karasievych, 2021; Demchenko, 2021; Kosholap, 2021; Ovcharuk, 2021) identify mathematical and methodical competence of primary school teachers and emphasize the importance of theoretical and practical use of mathematical knowledge, skills, and abilities when performing professional-pedagogical tasks. Others view mathematical competence as one’s integral ability that lies in acquiring mathematical knowledge and using it in professional activities (Borzenkova, 2007; Kazachek, 2010; Kudryavtsev, 2008). Also, there is a tendency towards general definitions of mathematical competence that can be projected onto any element of a teacher’s professional competence (Kudryavtsev, 2008; Miskova, 2013).

Significant guidelines for further inferences in the interpretation of this term are presented in the Standard of Higher Education for Specialty 013 “Primary Education” (a bachelor’s degree) in which mathematical competence is understood as the ability to apply specialized mathematical knowledge and skills that form the ideological, theoretical, and operational basis for the educational field “Mathematics”.

In the works of foreign scientists there are studies that study the formation of mathematical competence of the future primary school teacher by means of innovative technologies, the effectiveness of the introduction of modern technologies in the lessons of mathematics in today’s educational institutions. In scientific works D.Knüs (2014), M.Kepser (2009) revealed the essence of the concept of "competence of a modern teacher" and identified the problems of the present process of acquisition of
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mathematical competence in comparison with other European countries. M.Sorochinsky (2020); E. Barakhsanova (2020); E.Vlasova (2020); M. Prokopyev (2020); R. Boltuc (2015) proved that the teacher must have all the competencies necessary to master knowledge, but in the acquisition of mathematical - an even place takes ICT, drew attention to the need to borrow foreign experience in the perspective of the introduction of innovative technologies in primary education. A. Burnashev (2020) in his writings motivated the expansion and borrowing of the experience of implementing innovative and interactive technologies in elementary school in the study of mathematics. Foreign researchers E.Vlasova (2015), E. Barakhsanova (2015), S.Goncharova (2015), P.Aksyutin (2015), Z.Kuzin (2015) emphasized the need to implement e-learning to improve the educational process in the acquisition of mathematical competencies.

Based on the understanding of the views of scientists, guided by legislative documents in the field of education and taking as a basis the interpretation of mathematical competence of the future elementary school teacher, presented in the draft Standard of Higher Education in the specialty 013 "Elementary Education" (bachelor level) (manuscript) professional activities, in our study, the mathematical competence of the future primary school teacher is understood as a dynamic combination of professionally profiled mathematical knowledge, skills and practical skills, ways of thinking that are the basis of his ability to successfully solve professional problems in learning mathematics elementary school students.

Formation of competence of a future primary school teacher in the standardization of teacher education

In the conditions of renewal of professional pedagogical education there is an active discussion about the diversification of the teacher's work, expansion of his/her powers.

The inclusion of new activities in the model of primary school teacher training requires their comprehension and evidence. The most fruitful is the process of comprehension of pedagogical activity of the primary school teacher within the competence approach, Boltuc (2017).

The term "competence" (from Latin "competentia") literally means "correspondence, proportionality". In essence, it is a quantitative (proportionality) and qualitative (appropriateness) characteristic of general pedagogical and specialized subject knowledge that teachers should possess. In addition, there should be compliance with the current level of development of science, educational technology and commensurability of
this knowledge with the tasks that stand during the education and upbringing of children of the first and second changing ages. From the point of view of the competency approach the level of education of a teacher is determined by the ability to solve practical teaching and cognitive, educational, methodological and other problems on the basis of available knowledge, to use the obtained scientific knowledge in a particular activity. Innovative functions performed by primary school teachers give the profession a multidimensional character. The unidirectional professional competencies of an educator have now been replaced by multidirectional or multidimensional competencies (Sorochinsky, 2020; Barakhsanova, 2020; Vlasova, 2020; Prokopyev, 2020; Burnashev, 2020).

The relevance of this approach is explained by the needs of a developing society, the expansion of the cultural field of the profession, the multinational nature of social relations, value-oriented content of education and spiritual and moral education of new generations of children.

At the same time, the concept of "competence" is not new at all. Competent are professionally well-rounded people capable of properly performing their duties, functions. Such people are valued in any profession. The main thing in the teaching profession is to continually work on oneself first, and then to pass this skill on to children by explaining to them that "work" and "difficulties" to be overcome in learning are words of the same root, Melnyk (2021).

Work on ourselves is self-improvement, self-development, self-education, carried out through reading, writing, listening, speaking. This is the specificity of pedagogical activity: a teacher is a speech profession, therefore all kinds of speech activities are named here.

The two tools of pedagogical activity, language and the book (as the provider of language), make the teacher competent. And among all speech activities, reading is central because it is the only activity that belongs to man and distinguishes him from the animal world, Tkachuk (2021).

Improving one's own performance is the center of professional teacher training in the Department of Pedagogy and Methodology of Elementary Education.

**Identifying the Components of Future Primary School Teachers’ Mathematical Competence**

The meaning of the concept “mathematical competence of future primary school teachers” can be clarified through its structure. Therefore, it
is essential to analyze scientific approaches to identifying the components of this phenomenon.

According to Nizamieva (2010), the components of professional mathematical competence are motivational-axiological (motives for acquiring mathematical knowledge), cognitive (ways of comprehending the mathematical apparatus, as well as thinking abilities to do so) and conative (goal-setting and self-regulation skills). Zinenko (2009) holds similar views: motivational-axiological, cognitive, operational-technological, and reflexive.

Marko (2014) states that mathematical competence of primary school teachers as one’s integral qualitative characteristic that contains motivational-axiological, cognitive, operational, and reflexive-creative components characterizing psychological, theoretical, and practical readiness for pedagogical activities (teaching mathematics in primary school) and ensuring learning achievement among primary school pupils. The researcher especially focuses on the cognitive component as a set of general pedagogical, methodical, special (subject-specific) knowledge and axiological orientations based on pedagogical and mathematical values. Thus, one can conclude that Marko (2014) understands mathematical competence as methodical-mathematical competence of primary school teachers.

As noted by Razlivinskikh (2011, p. 26), mathematical competence is one’s integral characteristic that consists of such components as motivational-evaluative, cognitive-orientational, and operational-technological. These components contribute to the high effectiveness of mathematics teaching in primary school. In particular, the motivational-evaluative component is characterized by teacher students’ psychological readiness for professional activities. The cognitive-orientational component reflects pedagogical, methodical, special knowledge and axiological orientations based on pedagogical and mathematical values. The operational-technological component involves pedagogical, methodical, and special skills.

Borzenkova (2007, p. 17) identifies conceptual, reflective, and integrative-personal components. The conceptual component lies in understanding theoretical knowledge of mathematics, teaching methods, didactic, psychology, the basics of the future profession, as well as in developing professionally important skills. The reflexive component is determined through the ability to apply the acquired integrated knowledge in practice. The integrative-personal component involves the ability to integrate
different spheres of knowledge, and the striving for professional creativity, professional development, and research.

Gluzman (2010, pp. 149-152) singles out motivational-axiological, cognitive, activity-related, and reflexive-creative components. The motivational-axiological component contains a system of motives for professional activities. The cognitive component acts as a system of cognitive constructs, through the prism of which one evaluates the outside world, other people, and oneself, Gluzman (2010, p. 150). Unlike others, Gluzman (2010) highlights the activity-related component that enables motives for learning.

Similarly to O. Borzenkova (2007), N. Gluzman (2010, p. 52) includes the reflexive component to the structure of the methodological and mathematical competence of a future teacher, focusing on creativity, highlighting the reflexive-creative component. The researcher includes "knowledge and skills on the basics of innovative pedagogy, its social and scientific prerequisites, basic concepts, alternative approaches to the organization of teaching; knowledge and skills on pedagogical research methods; knowledge and skills on pedagogical communication; creative thinking, reflection control, individual and creative style of activity.

This research adheres to the following structure of mathematical competence of future primary school teachers: motivational-axiological, cognitive, activity-related, communicative, reflexive-creative.

The motivational-value component is focused on the axiological acquisition of knowledge from mathematical courses and continuing professional development. It consists of such factors as motives, values, and self-motivation in junior high school mathematics learning.

The cognitive component is viewed as a set of subject-specific knowledge and a system of cognitive constructs that ensure perception, understanding, and transfer of the acquired knowledge. It must be noted that this component relies on the basic concepts of the initial course in mathematics (multitude; mathematical statements and their structure; non-negative integers; geometric figures; values, etc.). The activity-related component manifests itself in solving mathematical problems, performing operations with algebraic expressions, applying logical operations in solving mathematical problems, constructing and proving statements. The communicative component lies in knowing special mathematical terms, transmitting mathematical information, and using verbal and non-verbal means of transmitting mathematical information. The reflective-creative
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component is revealed through the ability to reflect that is aimed at self-analysis of the acquired mathematical knowledge and skills for their practical use.

The explanatory note to the Standard of Higher Education for Specialty 013 “Primary Education” (a bachelor’s degree) contains the matrix of competencies defined by the Standard and conformity with appropriate descriptors. Thus, subject-specific mathematical competence includes relevant skills, knowledge, communication, autonomy, and responsibility.

Holders of a bachelor’s degree in primary education are to acquire the following knowledge:

- knowledge of a) conceptual approaches to constructing arithmetic of non-negative integers and developing the concepts of number, number systems, theory of divisibility and b) systems of homogeneous additive-scalar quantities, methods, and tools of their measurement;
- knowledge of a) basic algebraic concepts and rules of operating monomials, polynomials, and fractional-rational expressions and b) the relationships between expressions for variables;
- knowledge of a) properties and characteristics of polygons, polyhedra, and round shapes, b) types of plane mapping and their application; c) main characteristics of outstanding curves and surfaces;
- knowledge of a) theoretical foundations of Boolean algebras (set theory, mathematical logic), b) ways to define concepts and requirements for their correctness, c) types of mathematical statements and methods of their substantiation, d) basic schemes of correct and plausible reasoning, e) the essence of typical logical errors and the reasons for their occurrence;
- knowledge of a) the conceptual essence of identical transformations in mathematics, b) basic laws and rules of identical transformations of mathematical expressions, geometric figures, mathematical sentences.

Much attention is still paid to abilities as part of mathematical competence. They are the following:

- the ability to solve mathematical problems to clarify numerical dependencies and divisibility of expressions, find calculated (measuring) procedures, and establish the types of relationships between expressions;
- the ability to solve mathematical problems on identical transformations, the performance of operations with algebraic expressions, and the establishment of the minimum (maximum) values of the function on an interval by elementary methods;
- the ability to solve mathematical problems on constructing, cutting, and assembling figures, covering the plane with triangles and other figures, as well as on constructing scans of figures;

- the ability a) to apply set-theoretic and logical operations when solving mathematical problems, constructing statements and proving statements, b) use combinatorial concepts, rules, and formulas, the rules of inference (conclusion) and the basic schemes of correct reasoning in substantiating mathematical statements, c) construct logically correct mathematical sentences, d) establish the truth or falsity of statements, logical errors;

- the ability to perform identical transformations of mathematical expressions, mathematical sentences, geometric figures when solving mathematical problems.

The document largely focuses on the development of communication (the ability to use mathematical communication strategies to the extent sufficient for professional activities, broadcast to the essence of mathematical ideas, facts, methods of action according to age and cognitive characteristics of pupils; the ability to use mathematical concepts and facts in the course of mathematical reasoning; the ability to use verbal and nonverbal means of presenting mathematical information; the ability to use different ways of visualizing mathematical information), autonomy and responsibility (the ability to organize and manage mathematical activities, including when solving mathematical problems; the ability to reflect on mathematical and communicative activities; the ability to be responsible for reliability (correctness), validity and exhaustiveness of the results; the ability to engage in mathematical self-education in accordance with professional activities; the ability to shape ideas on mathematical worldview as the component of one’s general worldview; the ability to realize the role of mathematics in one’s intellectual development, in particular, in mental development of primary school pupils; the ability to understand the socio-cultural value of mathematical knowledge for today’s humanities-developmented society).

There are also studies on the structure of mathematical competence in the form of elements. For instance, Rakov (2005) distinguishes the following five elements in mathematical competence of future mathematics teachers: procedural, logical, technological, research, and methodological competences.

Let us try to briefly consider the most important aspects proposed by the researcher. The author understands procedural competence as the
ability of a future teacher to solve typical mathematical problems. Its base consists of the ability to:

a) the ability to practically use algorithms for solving typical problems;

b) the ability to recreate the context of the problems that might arise in individual and social practice and are reduced to typical problems;

c) the ability to systematize typical problems, find criteria of reducing problems to typical problems;

d) the ability to recognize a typical problem or reduce a specific problem to a typical one; e) the ability to use various information sources to find procedures for solving typical problems (textbooks, reference books, Internet resources) (Rakov, 2005, p. 5).

At the same time, Rakov (2005, pp. 5–6) considers logical competence as the ability to use the deductive method when analyzing statements. It lies in a) using the conceptual apparatus of deductive theories (definitions of concepts, their visual meaning, scope, properties, and boundaries; relationships between concepts), statements, predicates, logical operations, axioms, theorems, and b) proving theorems and specific examples to them. The researcher suggests that it is important “to build, improve and practically use one’s own system of mathematical representations based on the conceptual apparatus of deductive theories” (Rakov, 2005, pp. 5–6). Technological competence is understood as the knowledge of the latest ICTs to support mathematical activities; research competence – the ability to use relevant methods to study socially and individually significant problems with the help of ICTs and mathematical methods; methodological competence – the ability to assess the feasibility of using mathematical methods and ICTs to solve such problems (Rakov, 2005, pp. 5–6).

As defined by the Standard of Higher Education for Specialty 013 “Primary Education” (a bachelor’s degree), the elements of mathematical competence of future primary school teachers are as follows: arithmetic, logical, algebraic, geometric, as well as that of identical transformations of mathematical expressions (Ministerstvo osvity i nauky Ukrainy, 2021). They largely depend on the content of the mathematics course that makes it necessary to analyze regulatory programmes of this course.
The Structure of Mathematical Competence of Future Primary School Teachers

The authors of the article analyzed regulatory programmes and syllabi of the mathematics course in Ukrainian HEIs (Chernihiv National Pedagogical University named after Taras Shevchenko, South Ukrainian National Pedagogical University named after K. D. Ushynsky, Oleksandr Dovzhenko Hlukhiv National Pedagogical University, Vasyl Stefanyk Precarpathian National University, Donbas State Pedagogical University, Kherson State University, Pavlo Tychyna Uman State Pedagogical University) to develop the structure of mathematical competence of future primary school teachers, namely its content- and activity-related components. After studying the syllabi of the course, they traced a significant difference in academic workload among universities. For instance, Vasyl Stefanyk Precarpathian National University allocates 405 hours for the course, while Chernihiv National Pedagogical University named after Taras Shevchenko – only 90 hours.

There are also significant differences in the years of its study: in Year 1 (Chernihiv National Pedagogical University named after Taras Shevchenko); in Year 3 (South Ukrainian National Pedagogical University named after K. D. Ushynsky); in Years 1-3 (Vasyl Stefanyk Precarpathian National University).

A detailed analysis of the course content indicates that the topics for study are practically identical. Still, the sequence of studying the topics differs significantly.

All the analyzed syllabi clearly outline the knowledge of which topics future specialists are to acquire while studying the course:

a) the place of set theory in the system of mathematical concepts; definitions of subsets, unions, cross-sections, differences, Cartesian product of sets; laws and properties of operations on sets;

b) definition of correspondence between elements of two sets, binary relation onset, equivalence relation, order; the role of the expression concept in the system of concepts in mathematical logic; definition of logical operations and laws of these operations;

c) definition of expression, numerical expression, numerical equality and inequality, properties of valid numerical equations and inequalities, expression with a variable, equation and inequality with one variable, its root, set of solutions of equation and inequality; theorems on the equivalence of
equations and inequalities; definition of the system and the set of equations and inequalities;

d) definition of function, methods of setting functions; properties of a linear function, direct and inverse proportionality, quadratic function;

e) the essence of the axiomatic method of constructing the theory, Peano axiom; definition of a non-negative integer, arithmetic operations; laws of addition, multiplication;

f) definition of a numeral system; basic properties of positional decimal and non-decimal number systems; algorithms of actions in decimal and other number systems; g) definitions and properties of the divisibility relation, the divisibility theorem of sum, difference, and product; definition of prime and compound numbers; basic signs of divisibility; definitions and algorithms for finding GSD and LCM of two or more numbers;

h) definitions of a negative number, the set of integers, the modulus of a number, the rules of action on integers; definitions of fraction, equality of fractions, rational number as a class of equal fractions, operations on rational numbers, laws of addition and multiplication, properties of a set of rational numbers;

i) definitions of actual number, set of real numbers, its properties; definition of operations on real numbers, laws of addition, multiplication; definitions of approximate number, absolute and relative error, rules of rounding numbers, algorithms of actions on approximate values;

j) the essence of the axiomatic method of constructing geometry, axioms of school geometry, definitions, signs, and properties of geometric figures, including those studied in primary school; methods of geometric constructions with compasses and rulers;

k) definition of polyhedra: prisms, pyramids, regular polyhedra, their types; formulation of Euler’s theorem on the relationship between the number of vertices, faces, and edges of a convex polyhedron;

l) axiomatic properties of additive-scalar quantities; definition of the length of the segment, the area of the figure, the volume of the body as quantities, methods and units of measurement, the relationship between them, the formulas for the area and volume of geometric figures and bodies.

The program defines the skills that the student must acquire as a result of studying the course "Mathematics":

a) the ability to provide examples of numerical, point, and other sets and perform set-theoretic operations on finite and infinite sets;
b) the ability to provide examples of correspondences between sets, mutually unambiguous correspondence, binary relations, equivalence and order relations;

c) the ability to provide examples of statements and establish their truth;

d) the ability to determine the area of the truth of the predicate, use logical and mathematical symbolism, establish the existence of the relationship of sequence or equivalence between sentences, formulate them using the words “enough”, “necessary and sufficient”;

e) the ability to determine the scope and content of the concept, establish genus-species relations between concepts, provide examples of definitions and theorems of the school course in mathematics. Analyze the structure of the concept definition, highlight the condition and consequence of the theorem, prove it in one of the possible ways;

f) the ability to perform identical transformations of expressions, find a set of solutions of equations, including linear ones with one variable, both by theorems on the equivalence of equations and their consequences, and using the relationship between components and results of actions; inequalities and their systems, solve linear and quadratic equations, linear and quadratic inequalities, solve text problems by the method of equations; g) the ability to prove statements with the help of mathematical induction; h) the ability to find GSD and LCM of two or more numbers;

i) the ability to compare integers, fractions, reduce fractions, perform arithmetic operations on the set of integers and rational numbers;

j) the ability to prove properties and features of geometric figures, construct geometric shapes with a compass and a ruler. Draw images of polyhedrons and bodies of rotation in the plane.

According to Rakov’s classification (2005), standard mathematical competences are as follows:

1) practical competence – the ability to solve typical mathematical problems: to practically use an algorithm for solving typical problems, systematize typical tasks, find criteria for reducing tasks to typical, recognize a typical task or reduce it to a typical one, apply different information sources to find procedures for solving typical problems (textbook, reference book, Internet resources);

2) logical competence – the ability to use the deductive method to prove and refute statements: to practically use the conceptual apparatus of deductive theories (concepts, definitions; statements, axioms, theorems and
their proofs, counter-examples to theorems), recreate deductive proofs of the theorem and prove the correctness of procedures for solving typical problems, conduct deductive justifications for the correctness of problem-solving and search for logical errors in incorrect deductive reasoning, apply mathematical and logical symbols in practice.

Thus, one can see a difference in the distribution of academic workload, depending on the HEI, as well as the identical content of the topics for study.

Elements of Mathematical Competence of Future Primary School Teachers

Given the provisions of the Standard of Higher Education for Specialty 013 “Primary Education” (a bachelor’s degree) and scientific proposals and judgments, it is essential to specify the elements of mathematical competence of future primary school teachers. Thus, the elements of future teachers’ mathematical competence are defined as follows: arithmetic, algebraic, geometric, that of identical transformations of mathematical expressions, logical.

The arithmetic element is viewed as a set of mathematical knowledge, skills, and abilities in terms of the properties of numbers and performing actions on them, solving arithmetic problems. The basis of this component is:

a) definition of a non-negative integer as a common property of a class of equivalent finite sets, set-theoretic interpretation of “equal”, “less”, “more”; definitions of sum, difference, product, fraction, and conditions of their existence;

b) definitions and properties of divisibility relations, theorems on divisibility of sum, difference, and product; definition of simple and compound numbers; definitions and algorithms for finding GSD and LCM of two or more numbers;

c) definitions of a negative number, the set of integers, the modulus of a number, the rules of action on integers; definitions of fractions, equality of fractions, operations with rational numbers as a class of equal fractions, operations with rational numbers, laws of addition and multiplication, properties of a set of rational numbers; number, order, density;

d) definitions of actual number, set of real numbers, its properties; definition of operations on real numbers, laws of addition and
multiplication; definitions of approximate number, absolute and relative error, rules of rounding numbers;

e) the ability to compare integers, fractions, reduce fractions to a common denominator, perform operations of addition, subtraction, multiplication, and division on a set of integers, rational numbers, solve problems to find a fraction from a number and a fractional number.

The algebraic element is characterized by certain mathematical knowledge, skills, and abilities, including the concept of mathematical expression, equality, inequality, equation, dependence of the result of the arithmetic operation on the change of the component. The basis of this component is:

a) definition of expression, numerical expression, numerical equality and inequality, properties of true numerical equalities and inequalities, expression with a variable, equations and inequalities with one variable, its root, sets of solutions of equations and inequalities, theorems on equivalent equations and inequalities;

b) the ability to prove theorems on the equivalence of equations and inequalities, find many solutions of equations, including linear ones with one variable, both by theorems on the equivalence of equations and their consequences and using the relationship between components and results of actions; the ability solve linear and quadratic equations, linear and quadratic inequalities, and text problems by the method of equations;

c) the ability to find the domain and set of the function’s values, provide examples of functions, build function graphs, apply function properties to solve problems.

The geometric element is defined as certain mathematical knowledge, including the concept of lines and segments, drawing, and measuring the lengths of segments, acquaintance with polygons and circles, measuring the perimeter and areas of polygons, observing geometric bodies. The basis of this component is:

a) axioms of school geometry, definitions, signs, and properties of geometric figures, including those studied in primary school; methods of geometric constructions with compasses and rulers;

b) definition of polyhedra: prisms, pyramids, regular polyhedra, their types; formulation of Euler theorem on the relationship between the number of edges of a convex polyhedron;

c) definitions of the length of the segment, the area of the figure, the volume of the body as quantities, units of measurement, the relationship
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between them, the formula for measuring the area, the volume of geometric figures; d) the ability to prove properties, and features of geometric figures, construct geometric figures with a compass and a ruler, solve problems to calculate the elements of geometric figures;

d) the ability to practically measure quantities: length, area, volume, mass, as well as use formulas to calculate these quantities within geometric problems.

The element of identical transformations of mathematical expressions is a set of mathematical knowledge manifested in the ability to transform expressions while finding rational ways to solve them.

The logical element lies in using the deductive method to prove and disprove statements, as well as in performing logical operations while solving mathematical problems.

Conclusions

Sharing the opinion of N. Gluzman (2010) and I. Zimnaya (2004), a detailed analysis of relevant scientific literature makes it possible to consider mathematical competence in the context of its multi-levels: on the one hand, mathematical competence of future primary school teachers consists of motivational-axiological, cognitive, activity-related, communicative, and reflexive-creative components, on the other hand, it acts as a complex of such elements as arithmetic, algebraic, geometric, logical, that of identical transformations of mathematical expressions.

After considering regulatory documents and relevant studies, the authors of the article have defined the concept of “mathematical competence of future primary school teachers. Given the author’s definition of this concept, the elements and components of mathematical competence of future primary school teachers can be identified as follows: 1) arithmetic, algebraic, geometric, that of identical transformations of mathematical expressions, logical and 2) motivational-axiological, cognitive, activity-related, communicative, reflexive-creative.

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The author 1 carried out a critical review of existing publications on the problem of research, proved that the study of personal mathematical competence as a component of functional competence has been studied by foreign researchers and scientists and presented a description of the perspective of their research.
The author 2 presented in the article examples of determining the component composition of mathematical competence of the future primary school teacher.

The author 3 dealt with an important issue and laid down the structure of mathematical competence of a future primary school teacher.

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